THE *INTERIM* CORE OF A BAYESIAN PURE EXCHANGE ECONOMY

Tatsuro Ichiishi* **and Akira Yamazaki†**

September 2002

Abstract

Nonemptiness of the *interim* Bayesian incentive-compatible core of a Bayesian pure exchange economy is established.

*Department of Economics, The Ohio State University, and Graduate School of Economics, Hitotsubashi University. *Email*: ichiishi@economics.sbs.ohio-state.edu and ichiishi@econ.hit-u.ac.jp

†Graduate School of Economics, Hitotsubashi University. *Email*: yamazaki@econ.hit-u.ac.jp

1 Introduction

A *Bayesian pure exchange economy* is a pure exchange economy in which the consumers are asymmetrically endowed with information about their preference relations and initial endowments. A *strategy* in a Bayesian pure exchange economy is a plan which specifies a net trade for each state.[1] We adopt Harsanyi's (1967/68) type-profile approach, so a state is defined as a type profile. A strategy bundle is endogenously determined as a solution of the game played by the consumers. A strategy bundle is synonymously called a mechanism, so the theory endogenously determines a mechanism. After the consumers determine a mechanism, they execute it in the *interim* period, that is, each consumer chooses his particular net trade according to the plan when he knows his own type but not the others'. A solution is called *interim* if it is determined in the *interim* period. It is called *ex ante* if it is determined in the *ex ante* period.

Wilson (1978) initiated analysis of cooperative behavior in a Bayesian pure exchange economy, and proposed as *interim* solution concepts the *coarse core* concept and the *fine core* concept. He established nonemptiness of the coarse core, and also provided an example of an economy with an empty fine core.

Subsequent researchers paid particular attention to two basic conditions that a strategy needs to satisfy, in that the members of a coalition only design a strategy bundle which satisfies these two conditions. One condition, originally proposed by Radner (1968) in a different context, is the informational feasibility. It is the requirement that each plan be measurable with respect to the information structure available to the consumer at the time of his strategy-execution. Yannelis (1991) considered the *private information case* defined as the situation in which only the private information structure will be available to each consumer at the time of strategy-execution. A strategy bundle satisfying the informational feasibility for the private information case is called *private measurable*.

Another condition is the requirement that a strategy bundle be truthfully executed, namely d'Aspremont and Gérard-Varet's (1979) *Bayesian incentive compatibility*. Ichiishi and Idzik (1996) introduced this require-

[1] Some works define a plan as a function which specifies a consumption bundle for each state, or as a pair of a net-trade plan and a communication plan, but the present paper does not consider these strategies.

ment into a generalized core analysis, and defined the *ex ante Bayesian incentive-compatible core* as the core in the private information case in which each possible coalition designs a private measurable and Bayesian incentive-compatible strategy bundle in the *ex ante* period.

Our study object in the present paper is the *interim* analogue of the *ex ante* Bayesian incentive-compatible core. Besides the obvious merit of satisfying the two basic conditions, it enjoys the strong properties that it is essentially as fine as the fine core, and that the grand coalition and all possible blocking coalitions are treated symmetrically (a property which the coarse core and the fine core fail to satisfy). Here, by saying symmetric treatment, we are expressing our understanding that consumers form either the grand coalition or any possible blocking coalition in the same way, in order to jointly determine a strategy bundle for pursuit of their self-interest. Our main result is that a Bayesian pure exchange economy with concave and weakly monotone state-dependent von Neumann-Morgenstern utility functions has a nonempty *interim* Bayesian incentive-compatible core.

We remark, however, that this positive result is unlikely to be extended beyond the pure exchange economy.

For a survey of the works on *ex ante* cores of a Bayesian pure exchange economy, see, e.g., Ichiishi and Yamazaki (2002, subsection 3.5).

2 Model and Result

We study a pure exchange economy with l commodities and n consumers in which consumers are asymmetrically endowed with information about the data of the economy. Let N be the set of n consumers, and let $\mathcal{N}$ $(:= 2^N \setminus \{\emptyset\})$ be the family of nonempty coalitions. In accordance with Harsanyi's (1967/1968)'s framework of the Bayesian game, let T^j be a finite set of consumer j's types. The type-profile space for each coalition S is then given as $T^S := \prod_{j \in S} T^j$. Set for simplicity, $T := T^N$. A member of T is synonymously called a type profile or a state. The *interim* period is defined here as the period during which each consumer j knows his true type $t^j \in T^j$ but not the others'. His private information structure, denoted by $\mathcal{T}^j$, is the algebra on T generated by the sets, $\{t^j\} \times T^{N \setminus \{j\}}$, $t^j \in T^j$.

Given type profile t, each consumer is characterized by his consumption set $\mathbf{R}^l_+$, his state-dependent von Neumann-Morgenstern utility function

$u^j(\cdot, t) : \mathbf{R}^l_+ \to \mathbf{R}$, his initial endowment vector $e^j(t^j) \in \mathbf{R}^l_+$, and his *interim* probability $\pi^j(\cdot \mid t^j)$ on T given t^j. We are assuming here that the initial endowment function e^j depends only on t^j, in other words, e^j (when viewed as a function on T) is $\mathcal{T}^j$-measurable. If the consumers commonly hold an objective *ex ante* probability π on T, consumer j's *interim* probability is given by the Bayes' rule,

$$\pi^j(t \mid \bar{t}^j) := \begin{cases} \frac{\pi(t)}{\sum\limits_{t^{N\setminus\{j\}} \in T^{N\setminus\{j\}}} \pi(\bar{t}^j, t^{N\setminus\{j\}})}, & \text{if } t^j = \bar{t}^j, \\ 0, & \text{otherwise,} \end{cases}$$

but our present framework allows for subjective probabilities, and even inconsistency among *interim* probabilities (in that there is no *ex ante* probability to which the Bayes' rule can be applied in order to derive the *interim* probabilities).

DEFINITION 2.1 A *Bayesian pure exchange economy*

$$\mathcal{E}_{pe} := \{T^j,\ \mathbf{R}^l_+,\ u^j,\ e^j,\ \{\pi^j(\cdot \mid t^j)\}_{t^j \in T^j}\}_{j \in N}$$

is an economy with l commodities, where N is a consumer set, and for each consumer j, T^j is his type set, $\mathbf{R}^l_+$ is his consumption set, $u^j : \mathbf{R}^l_+ \times T \to \mathbf{R}$ is his type-profile-dependent von Neumann-Morgenstern utility function, $e^j : T \to \mathbf{R}^l_+$ is his initial endowment function, postulated to be $\mathcal{T}^j$-measurable, and $\pi^j(\cdot \mid t^j)$ is his *interim* probability on the states T given t^j.

We analyze consumers' cooperative behavior during the *interim* period. When type profile t prevails, consumer j chooses his net trade vector $z^j(t)$ ($\in \mathbf{R}^l$). Any set S of consumers can come together and jointly schedule a net-trade plan $z^S := \{z^j\}_{j \in S}$, so that the plan is *individually feasible*,

$$\forall\ t \in T : \ \ \forall\ j \in S : \ \ z^j(t) + e^j(t^j) \in \mathbf{R}^l_+, \tag{1}$$

and is *attainable*,

$$\forall\ t \in T : \ \ \sum_{j \in S} z^j(t) \leq \mathbf{0}. \tag{2}$$

Consumer j's net-trade plan $z^j : T \to \mathbf{R}^l$ is his strategy.[2] Denote by F^S the set of all feasible strategy bundles for coalition S:

$$F^S := \{z^S : T \to \mathbf{R}^{l \cdot \# S} \mid z^S \text{ satisfies (1) and (2)}\}.$$

Not all plans in F^S can actually be chosen. We discuss two basic conditions that a plan has to satisfy. We are studying consumers' behavior in (what the literature has called) the private information case, that is the private information structure $\mathcal{T}^j$ is the information structure available to j at the time of j's action (choice of net trade). The first condition, therefore, stipulates that a plan be informationally feasible in this case; to be precise,

$$\forall\, j \in S : \quad z^j \text{ is } \mathcal{T}^j\text{-measurable.} \tag{3}$$

A plan z^S satisfying condition (3) is called private measurable. Denote by F'^S the set of all feasible, private-measurable strategy bundles for coalition S:

$$F'^S := \{z^S \in F^S \mid z^S \text{ satisfies (3)}\}.$$

The second condition pertains to the feasibility of execution of strategy bundles viewed as "contracts" made within a coalition. Suppose that the members of coalition S agree to take a strategy bundle $z^S \in F'^S$. Let $\bar{t}^j$ be consumer j's true type. If he makes a choice according to the agreement, his *interim* expected utility given his true type is

$$Eu^j(z^j + e^j \mid \bar{t}^j) := \sum_{t \in T} u^j\left(z^j(\bar{t}^j) + e^j(\bar{t}^j), t\right) \pi^j(t \mid \bar{t}^j).$$

In the private information case, however, he can make any choice $c^j \in z^j(T^j) \setminus \{z^j(\bar{t}^j)\}$ contrary to the agreement, yet his colleagues $S \setminus \{j\}$ cannot catch this betraying act, being led to believe that j's true type were in the event $(z^j)^{-1}(c^j)$. If j makes such a choice, his *interim* expected utility given his true type is

$$Eu^j(c^j + e^j \mid \bar{t}^j) := \sum_{t \in T} u^j\left(c^j + e^j(\bar{t}^j), t\right) \pi^j(t \mid \bar{t}^j).$$

[2] It is important that j's strategy is his net-trade plan z^j. Some models postulate that j's strategy is his consumption plan, $t \mapsto z^j(t) + e^j(t^j)$, but we cannot obtain the existence result (the main result of this paper) with this alternative definition of strategy.

The members of the coalition decide on plan z^S, in order to realize the choice bundle $z^S(t)$ at each possible type profile t. If member j does not act according to the agreed plan, taking advantage of his private information, the required outcome $z^S(\bar{t})$ cannot be achieved and the purpose of the coalition formation is not fulfilled. To guarantee realization of the exact execution of an agreement, the members of the coalition agree only to those plans z^S which nobody has an incentive to act contrary to. In short, they agree only to the Bayesian incentive-compatible plans. To be precise, the Bayesian incentive compatibility is defined as:

$$\forall\ t^j, \tilde{t}^j \in T^j : \quad Eu^j(z^j + e^j \mid t^j) \geq Eu^j(z^j(\tilde{t}^j) + e^j \mid t^j). \tag{4}$$

Notice that unlike Vohra (1999), there is no mediator in our model, since we are modelling the reality in which the members of a coalition seldom consult with an outsider (mediator) in carrying out their own agreement.

Let $\hat{F}^S$ be the set of all feasible, private measurable and Bayesian incentive-compatible strategy bundles:

$$\hat{F}^S := \{z^S \in F'^S \mid z^S \text{ satisfies (4)}\}.$$

We remark that $\mathbf{0} \in \hat{F}^S$, in particular each set $\hat{F}^S$ is nonempty.

The consumers play a cooperative game during the *interim* period. An *interim* Bayesian incentive-compatible core net-trade plan is a strategy bundle of the grand coalition N upon which no coalition can improve regardless of any possible type profile:

DEFINITION 2.2 An *interim Bayesian incentive-compatible core net-trade plan* of a Bayesian pure exchange economy $\mathcal{E}_{pe}$ is a strategy bundle z^* such that

(i) $$z^* \in \hat{F}^N; \quad \text{and}$$

(ii) it is not true that

$$\begin{aligned} &\exists\ S \in \mathcal{N} : \exists\ t^S \in T^S : \exists\ z^S \in \hat{F}^S : \\ &\quad \forall\ j \in S : \quad Eu^j(z^j + e^j \mid t^j) > Eu^j(z^{*j} + e^j \mid t^j). \end{aligned}$$

Condition (i) in definition 2.2 is the feasibility of the plan z^* (via the grand coalition), and condition (ii) is the coalitional stability condition.

The main result of this paper is the following existence theorem. A function $f : \mathbf{R}^l_+ \to \mathbf{R}$ is called weakly monotone, if

$$[c, c' \in \mathbf{R}^l_+,\ c \leq c'] \ \Rightarrow\ f(c) \leq f(c').$$

THEOREM 2.3 *Let $\mathcal{E}_{pe}$ be a Bayesian pure exchange economy. Assume for each consumer j that his von Neumann-Morgenstern utility function $u^j(\cdot, t)$ is continuous, concave, and weakly monotone in $\mathbf{R}^l_+$ for every $t \in T$. Then there exists an interim Bayesian incentive-compatible net-trade plan.*

REMARK 2.4 In our proof of theorem 2.3 (section 3), we show the existence of an *interim* Bayesian incentive-compatible core net-trade plan z^* which is exactly attained in the grand coalition, that is,

$$\sum_{j \in N} z^{*j}(t) = \mathbf{0}, \quad \text{for every } t \in T,$$

while allowing for the broad range of strategies (the *weak* inequality (2)) for all possible blocking coalitions. □

REMARK 2.5 Vohra (1999)'s example of an empty Bayesian incentive-compatible coarse core is crucially based on his postulate that for a plan z^S of coalition S, each strategy z^j is a function of $t^S \in T^S$, rather than of $t^j \in T^j$. Vohra's setup requires the presence of a mediator who, by collecting private information, enlarges the set of possible blocking strategies, thereby making coalition-formation easier. □

3 Proof of Theorem 2.3

An *interim private core net-trade plan* of a Bayesian pure exchange economy $\mathcal{E}_{pe}$ is a strategy bundle z^* that satisfies all the conditions of definition 2.2 (*interim* Bayesian incentive-compatible core net-trade plan) in which the sets $\hat{F}^S$ are replaced by F'^S, $S \in \mathcal{N}$.

The idea of our proof of theorem 2.3 is to (1) construct an auxiliary economy in agent form $\mathcal{E}_a$ associated with the economy $\mathcal{E}_{pe}$, (2) prove the existence of a core net trade z^* of $\mathcal{E}_a$, by applying Scarf's (1967) theorem on a balanced game, (3) show that z^* is also a private core net-trade plan of $\mathcal{E}_{pe}$, (4) show the existence of a private core net-trade plan z^{**} of $\mathcal{E}_{pe}$ for which $\sum_{j\in N} z^{**j}(t) \equiv \mathbf{0}$, by applying Ichiishi and Radner (1999, lemma 6.3, page 330), and (5) show that z^{**} is also Bayesian incentive-compatible, by applying Hahn and Yannelis (1997, proposition 6.10, page 401).

Selten (1975) defined the game in agent normal form associated with an extensive game. An agent controls only one information set (so different information sets of a player are controlled by different agents), his pure strategy set is the same as the choice set for the given information set, and his utility function is the same as the utility function of the player who controls the information set. We apply this idea to the present model of Bayesian pure exchange economy. In Selten's work on equilibrium refinement, the concept of agent's conditional expected utility given his information set plays a central role. In the present work on an *interim* solution, the concept of agent's conditional expected utility given his type plays a central role; roughly stated, realization of a type determines an information set.[3]

An *agent* of economy $\mathcal{E}_{pe}$ is defined as a consumer together with his type, (j, t^j); denote by A the set of all agents,

$$A := \{(j, t^j) \mid j \in N, t^j \in T^j\}.$$

An economy in agent form $\mathcal{E}_a$ is defined as a particular static pure exchange economy:

DEFINITION 3.1 The *economy in agent form* associated with Bayesian pure exchange economy $\mathcal{E}_{pe}$ is a static pure exchange economy,

$$\mathcal{E}_a := \{\mathbf{R}^l_+,\ Eu^j(\cdot \mid t^j),\ e^j(t^j)\}_{(j,t^j)\in A},$$

with l commodities, where A is the agent set, and for each agent (j, t^j), $\mathbf{R}^l_+$ is his consumption set, $Eu^j(\cdot \mid t^j) : \mathbf{R}^l_+ \to \mathbf{R}$ is his utility function, and $e^j(t^j)$ $(\in \mathbf{R}^l_+)$ is his initial endowment vector.

The attainability condition for economy in agent form $\mathcal{E}_a$ is, however, defined differently from the conventional one: A net trade bundle $\{z^{(j,t^j)}\}_{(j,t^j)\in A}$ of $\mathcal{E}_a$ is called *feasible in the grand coalition*, if

$$\forall\ (j, t^j) \in A : \qquad z^{(j,t^j)} + e^j(t^j) \in \mathbf{R}^l_+, \tag{5}$$

$$\forall\ t \in T : \qquad \sum_{j\in N} z^{(j,t^j)} \leq \mathbf{0}. \tag{6}$$

[3] To be precise, we are making only an analogy to Selten's procedure. In fact, unlike in the extensive game, coalition S's feasible choice set is not the cartesian product of individual consumers' feasible choice sets in the pure exchange economy.

Denote by G the set of all feasible strategy bundles in the grand coalition of $\mathcal{E}_a$:

$$G := \left\{ \{z^{(j,t^j)}\}_{(j,t^j)\in A} \;\middle|\; \{z^{(j,t^j)}\}_{(j,t^j)\in A} \text{ satisfies (5) and (6)} \right\}.$$

A feasible, private-measurable strategy bundle of $\mathcal{E}_{pe}$ is characterized as a feasible strategy bundle in the grand coalition of $\mathcal{E}_a$. Indeed, $z \in F'^N$ corresponds to $\{z^j(t^j)\}_{(j,t^j)\in A} \in G$.

We next re-define the *interim* coalitional stability condition for the private core of $\mathcal{E}_{pe}$, using the concept of agents. An *admissible blocking coalition* in economy in agent form $\mathcal{E}_a$ is a coalition of agents in which at most one agent represents each consumer; denote by $\mathcal{B}_0$ the family of all admissible blocking coalitions,

$$\mathcal{B}_0 := \{B \subset A \mid [(i,t^i),(j,t^j) \in B, t^i \neq t^j] \Rightarrow i \neq j\}.$$

For $B \in \mathcal{B}_0$, let $S(B)$ be the set of those consumers represented by the agents B,

$$S(B) := \{j \in N \mid \exists\, t^j \in T^j : (j,t^j) \in B\}.$$

Then, the coalitional stability condition on a strategy bundle $\{z^{*(j,t^j)}\}_{(j,t^j)\in A}$ of $\mathcal{E}_a$ is defined as follows; it is seemingly different from the conventional coalitional stability condition.

$$\begin{aligned}&\neg\, \exists\, B \in \mathcal{B}_0 : \exists\, z^{S(B)} \in F'^{S(B)} : \\ &\quad \forall\, (j,t^j) \in B : \;\; Eu^j(z^j + e^j \mid t^j) > Eu^j(z^{*(j,t^j)} + e^j \mid t^j).\end{aligned}$$

Interim blocking with a feasible, private-measurable strategy bundle of coalition S in $\mathcal{E}_{pe}$ is characterized as blocking in $\mathcal{E}_a$ with a strategy bundle in $F'^{S(B)}$ for some $B \in \mathcal{B}_0$ for which $S = S(B)$.

We thus have the following re-formulation:

PROPOSITION 3.2 *A net-trade plan z^* is an interim private core net-trade plan in Bayesian pure exchange economy $\mathcal{E}_{pe}$, if and only if the net trade bundle $\{z^{*j}(t^j)\}_{(j,t^j)\in A}$ is a core net trade bundle of the economy in agent form $\mathcal{E}_a$ associated with $\mathcal{E}_{pe}$.*

To apply Scarf's theorem for nonemptiness of the core, define the non-side-payment game (V, H) associated with $\mathcal{E}_a$:

$$\begin{aligned} V(B) &:= \left\{ u \in \mathbf{R}^A \;\middle|\; \begin{array}{l} \exists\, z^{S(B)} \in F'^{S(B)} : \forall\, (j, t^j) \in B : \\ u_{(j,t^j)} \leq Eu^j(z^j(t^j) + e^j(t^j) \mid t^j) \end{array} \right\} \text{ if } B \in \mathcal{B}_0, \\ V(B) &:= \emptyset \text{ if } B \notin \mathcal{B}_0, \\ H &:= \left\{ u \in \mathbf{R}^A \;\middle|\; \begin{array}{l} \exists\, \{z^{(j,t^j)}\}_{(j,t^j) \in A} \in G : \forall\, (j, t^j) \in A : \\ u_{(j,t^j)} \leq Eu^j(z^{(j,t^j)} + e^j(t^j) \mid t^j) \end{array} \right\}. \end{aligned}$$

The sets $V(B)$ are the sets of utility allocations attainable in blocking coalitions $B \in 2^A \setminus \{\emptyset\}$. The set H is the set of utility allocations attainable in the grand coalition A.

Let $\chi_B \in \mathbf{R}^A$ be the characteristic vector of subset B of A. A subfamily $\mathcal{B}$ of $\mathcal{B}_0$ is called *balanced*, if there exists associated balancing coefficients, $\{\lambda_B\}_{B \in \mathcal{B}}$, i.e.,

$$\lambda_B \geq 0, \; \sum_{B \in \mathcal{B}} \lambda_B \chi_B = \chi_A.$$

Game (V, H) is called *balanced*, if for every balanced subfamily $\mathcal{B}$ of $\mathcal{B}_0$,

$$\bigcap_{B \in \mathcal{B}} V(B) \subset H.$$

Proof of Theorem 2.3. We first show that the non-side-payment game (V, H) derived from the economy in agent form $\mathcal{E}_a$ is balanced. Indeed, this is done in the same way as Scarf (1967) has shown that the game associated with a pure exchange economy is balanced: Choose any balanced subfamily $\mathcal{B}$ of $\mathcal{B}_0$, with the associated balancing coefficients $\{\lambda_B\}_{B \in \mathcal{B}}$, and any $u \in \bigcap_{B \in \mathcal{B}} V(B)$. For each $B \in \mathcal{B}$, there exists $\{z^{(B,j))}\}_{j \in S(B)} \in F'^{S(B)}$ which gives rise to $\{u_{(j,t^j)}\}_{(j,t^j) \in B}$. Define $\bar{z}^N$ by $\bar{z}^j(t^j) := \sum_{B \in \mathcal{B} : B \ni (j,t^j)} \lambda_B z^{(B,j)}(t^j)$. It is easy to show that $\bar{z}^N$ gives rise to u, and that $\sum_{j \in N} \bar{z}^j(t^j) \leq \mathbf{0}$ for every t. Consequently, $u \in H$, so game (V, H) is balanced.

By Scarf's (1967) theorem, there exists a core net trade bundle z^* of $\mathcal{E}_a$. By proposition 3.2, it is also a private core plan of $\mathcal{E}_{pe}$.

By Ichiishi and Radner (1999, lemma 6.3, page 330), there exists private-measurable plan $z^{**} : T \to \mathbf{R}^{l \cdot \#N}$ such that

$$\begin{aligned} \forall\, t \in T : \qquad & \forall\, j \in N : \; z^{*j}(t^j) \leq z^{**j}(t^j), \\ & \sum_{j \in N} z^{**j}(t^j) = \mathbf{0}. \end{aligned}$$

In view of the weak monotonicity of each $u^j(\cdot, t)$, z^{**} is also a private core plan of $\mathcal{E}_{pe}$.

According to the plan z^{**}, total use of resources is exactly the same as the total initial resources for every $t \in T$. Therefore, by Hahn and Yannelis (1997, proposition 6.10, page 401), z^{**} is Bayesian incentive compatible. $\square$

4 Information-Revelation through Coalition Formation

Consumer j's very act of joining a coalition may reveal (a part of) his private information, so condition (ii) of definition 2.2 (coalitional stability condition for a *interim* Bayesian incentive-compatible core net-trade plan) may be inappropriate. This observation has been known among the researchers of the present research area as a folklore (see, e.g., Ichiishi and Yamazaki (2002, subsection 3.2.2)).

The following proposition clarifies a rather specific case in which such information revelation does not influence the consumers' final decision on coalition formation; essentially, it is the situation in which information on the other consumers' types $T^{S\setminus\{j\}}$ is irrelevant to j's *interim* expected utility.

PROPOSITION 4.1 *Let $j \in S$. Assume the $\mathcal{T}^j$-measurability of j's strategies. Assume also the no-externality, that is, u^j depends only on $(c^j, t^j) \in \mathbf{R}^l_+ \times T^j$. Then, revelation of information on $T^{S\setminus\{j\}}$ does not affect j's decision in regard to joining coalition S.*

Proof. Let $z^S \in F'^S$. By the no externality,

$$Eu^j(z^j + e^j \mid t^j) = u^j(z^j(t^j) + e^j(t^j), t^j).$$

Suppose that willingness of the members $S \setminus \{j\}$ to choose z^S reveals information A ($\subset T^{S\times\{j\}}$). Then,

$$Eu^j(z^j + e^j \mid t^j, A) = u^j(z^j(t^j) + e^j(t^j), t^j).$$

So, consumer j's decision in regard to whether or not to accept z^S instead of any other plan $z^{\dagger S}$ does not change after obtaining information A. $\square$

References

d'Aspremont, C., and L.-A. Gérard-Varet (1979): "Incentives and incomplete information," *Journal of Public Economics* **11**, 22-45.

Hahn, G., and N. C. Yannelis (1997): "Efficiency and incentive compatibility in differential information economies," *Economic theory* **10**, 383-411.

Harsanyi, J. C. (1967/1968): "Games with incomplete information played by 'Bayesian' players," *Management Science: Theory* **14**, 159-182 (Part I), 320-334 (Part II), 486-502 (Part III).

Ichiishi T., and A. Idzik (1996): "Bayesian cooperative choice of strategies," *International Journal of Game Theory* **25**, 455-473

Ichiishi, T., and R. Radner (1999): "A profit-center game with incomplete information," *Review of Economic Design* **4**, 307-343.

Ichiishi, T., and A. Yamazaki (2002): "Preliminary results for cooperative extensions of the Bayesian game," Discussion Paper #2001-9, Graduate School of Economics, Hitotsubashi University.

Radner, R. (1968): "Competitive equilibrium under uncertainty," *Econometrica* **36**, 31-58.

Selten, R. (1975): "Reexamination of the Perfectness Concepts for Equilibrium Points in Extensive Games," *International Journal of Game Theory* **4**, 25-55.

Vohra, Rajiv (1999): "Incomplete information, incentive compatibility, and the core," *Journal of Economic Theory* **86**, 123-147.

Wilson, R. (1978): "Information efficiency, and the core of an economy," *Econometrica* **46**, 807-816.

Yamazaki, A., and T. Ichiishi (in preparation): "Incentive compatibility and the core of a large economy with differential information."

Yannelis, N. C. (1991): "The core of an economy with differential information," *Economic Theory* **1**, 183-198.